

Microscopic Calculation of Spin Torques in Disordered Ferromagnets

Hiroshi Kohno*

Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan

Gen Tatara

*Graduate School of Science, Tokyo Metropolitan University, Hachioji, Tokyo 192-0397, Japan
PRESTO, JST, Kawaguchi, Saitama 332-0012, Japan*

Junya Shibata

*RIKEN-FRS, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
(Dated: February 3, 2008)*

Effects of conduction electrons on magnetization dynamics, represented by spin torques, are calculated microscopically in the first order in spatial gradient and time derivative of magnetization. Special attention is paid to the so-called β -term and the Gilbert damping, α , in the presence of electrons' spin-relaxation processes, which are modeled by quenched magnetic (and spin-orbit) impurities. The obtained results such as $\alpha \neq \beta$ hold for localized as well as itinerant ferromagnetism.

PACS numbers: 72.25.Ba, 72.15.Gd, 72.25.Rb

Recent activity in spintronics to manipulate nanoscale magnetization, such as magnetization reversal, by electric current started with the concept of spin-transfer effect introduced by Slonczewski [1] and Berger [2]. When a current flows in a background magnetization \mathbf{n} varying in space, a slight directional mismatch (with \mathbf{n}) arises in the electron spin polarization such that spin angular momentum of a spin-polarized current (spin current) is transferred to the magnetization. This effect is expressed as a torque on \mathbf{n} , called spin-transfer torque; for slowly-varying \mathbf{n} , it takes the form, $-(\mathbf{v}_s \cdot \nabla)\mathbf{n}$, where \mathbf{v}_s is a velocity characterizing the spin-transfer rate [3, 4, 5, 6, 7].

Existence of another type of current-induced spin torque, of the form, $-\beta\mathbf{n} \times (\mathbf{v}_s \cdot \nabla)\mathbf{n}$, was noted recently [8, 9, 10]. This torque is perpendicular to the spin-transfer torque, and is called nonadiabatic torque or the β -term. Zhang and Li showed on phenomenological grounds that it arises as a result of spin relaxation of conduction electrons [8]. Barnes and Maekawa suggested its close connection to the Gilbert damping, α , and proposed a relation $\alpha = \beta$ [10]. While the importance of the β -term in magnetization dynamics is now well-recognized [8, 9, 10, 11], its microscopic derivation is a current issue.

Similar torques are known also in multilayer [12, 13], domain wall [14, 15, 16], and other [17, 18] systems. For example, for a rigid domain wall, a torque which acts as a force and may thus be called momentum-transfer torque, was noted [14, 16]. This is a spatially oscillating torque due to electron reflection, found by Waintal and Viret [15], which has the same algebraic form as the β -term but is spatially nonlocal [19]. This torque can be important for magnetic configurations varying rapidly in space whereas the β -term is relevant to slowly-varying configurations. We focus on the β -term in this Letter.

Very recently, a microscopic study of this subject was undertaken by Tserkovnyak, Brataas and Bauer (TBB)

based on the Boltzmann equation [20]. They showed that $\alpha = \beta$ for an itinerant 'single-band' ferromagnet, and emphasized its peculiar magnetization dynamics. Their analysis is, however, still phenomenological as to the spin-relaxation term which was introduced by hand.

In this Letter, we present a microscopic calculation of spin torques, especially the β -term and the Gilbert damping, in a localized (s - d) as well as an itinerant (Stoner) models for ferromagnetism. We introduce spin-relaxation processes into the (conducting) electron system by magnetic (and spin-orbit) impurities causing spin-flip and spin-dependent scatterings. Our result shows that both transverse and longitudinal relaxation processes contribute to α and β , and that $\alpha \neq \beta$ in general.

We first consider a localized (s - d) model. It consists of localized d -spins, \mathbf{S} , and conducting s -electrons, which are coupled *via* the s - d exchange interaction

$$H_{sd} = -M \int d\mathbf{r} \mathbf{n}(\mathbf{r}) \cdot \hat{\boldsymbol{\sigma}}(\mathbf{r}). \quad (1)$$

Here we put $\mathbf{S} = S\mathbf{n}$ with a unit vector \mathbf{n} pointing in the direction of spin [21], $\hat{\boldsymbol{\sigma}}(\mathbf{r}) = c^\dagger(\mathbf{r})\boldsymbol{\sigma}c(\mathbf{r})$ represents (twice) the s -electron spin density, with $c^\dagger = (c^\dagger_\uparrow, c^\dagger_\downarrow)$ being electron creation operators, $\boldsymbol{\sigma}$ the Pauli spin-matrix vector, and $M = J_{sd}S$ with J_{sd} being the s - d exchange coupling constant. The total Hamiltonian of the system is given by $H_{\text{tot}} = H_S + H_{\text{el}} + H_{sd}$, where H_S and H_{el} are for localized d -spins and s -electrons, respectively.

The dynamics of magnetization, $-\dot{\mathbf{n}}$ [21], will be described by the Landau-Lifshitz-Gilbert (LLG) equation

$$\dot{\mathbf{n}} = \gamma_0 \mathbf{H}_{\text{eff}} \times \mathbf{n} + \alpha_0 \dot{\mathbf{n}} \times \mathbf{n} + \mathbf{t}'_{\text{el}}, \quad (2)$$

where $\gamma_0 \mathbf{H}_{\text{eff}}$ and α_0 are an effective field and a Gilbert damping constant, respectively, both coming from H_S and not from processes involving s -electrons. Effects of

conducting s -electrons are contained in the spin torque

$$\mathbf{t}_{\text{el}}(\mathbf{r}) \equiv \frac{\hbar S}{a^3} \mathbf{t}'_{\text{el}}(\mathbf{r}) = M \mathbf{n}(\mathbf{r}) \times \langle \hat{\boldsymbol{\sigma}}(\mathbf{r}) \rangle_{\text{n.e.}}, \quad (3)$$

which comes from H_{sd} . (Here a^3 is the volume per localized spin.) The calculation of spin torque is thus equivalent to that of s -electron spin polarization, $\langle \hat{\boldsymbol{\sigma}}(\mathbf{r}) \rangle_{\text{n.e.}}$, or precisely, its perpendicular projection $\langle \hat{\boldsymbol{\sigma}}_{\perp}(\mathbf{r}) \rangle_{\text{n.e.}}$ to \mathbf{n} [22], in such nonequilibrium states with current flow and spatially varying magnetization (for the β -term), or with time-dependent magnetization (for Gilbert damping). Here and hereafter, $\langle \cdots \rangle_{\text{n.e.}}$ represents statistical average in such nonequilibrium states.

Generally, the spin torque is expressed as

$$\mathbf{t}_{\text{el}} = a_0 \dot{\mathbf{n}} + (\mathbf{a} \cdot \nabla) \mathbf{n} + b_0 (\mathbf{n} \times \dot{\mathbf{n}}) + \mathbf{n} \times (\mathbf{b} \cdot \nabla) \mathbf{n} \quad (4)$$

in the first order in time derivative and spatial gradients. The \mathbf{a} -term includes the spin-transfer torque, $-(\mathbf{v}_s \cdot \nabla) \mathbf{n}$, where $\mathbf{v}_s = -(a^3/2eS)\mathbf{j}_s$ ($e > 0$ being the elementary charge) [21] is a quantity which may be called as the spin-transfer velocity. (The a_0 -term just renormalizes the magnitude of spin, $S \rightarrow S - (a^3/\hbar)a_0$ on the left-hand side of the LLG equation, or in front of the kinetic (“Berry phase”) term in the spin Lagrangian [7].) The \mathbf{b} -term is the central issue in the present study. We also focus on b_0 , which gives damping of the Gilbert type, with dimensionless damping constant $\alpha = -(a^3/\hbar S)b_0$. The \mathbf{b} -term is expressed as $-\beta \mathbf{n} \times (\mathbf{v}_s \cdot \nabla) \mathbf{n}$ (in \mathbf{t}'_{el}) [21] in the literature [9, 10, 20], which defines the constant β .

The spin torque (4) corresponds to the s -electron spin polarization $\langle \hat{\boldsymbol{\sigma}}_{\perp} \rangle_{\text{n.e.}}$ [22] given by

$$\frac{1}{M} [b_0 \dot{\mathbf{n}} + (\mathbf{b} \cdot \nabla) \mathbf{n} - a_0 (\mathbf{n} \times \dot{\mathbf{n}}) - \mathbf{n} \times (\mathbf{a} \cdot \nabla) \mathbf{n}]. \quad (5)$$

To calculate the coefficients, a_{μ} and b_{μ} , we follow TBB [20] (see also Ref.[23]) and consider a small transverse fluctuation, $\mathbf{u} = (u^x, u^y, 0)$, $|\mathbf{u}| \ll 1$, around a uniformly magnetized state, $\mathbf{n} = \hat{z}$, such that $\mathbf{n} = \hat{z} + \mathbf{u}$. In the ‘unperturbed’ state, $\mathbf{n} = \hat{z}$, the s -electrons are described by the Hamiltonian

$$\mathcal{H}_0 = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \varepsilon_{\text{F}\sigma}) c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + V_{\text{imp}}, \quad (6)$$

and have a spin polarization $\langle \hat{\boldsymbol{\sigma}} \rangle_0 = \rho_s \hat{z}$. Here $\rho_s = n_{\uparrow} - n_{\downarrow}$, $n_{\sigma} = k_{\text{F}\sigma}^3/6\pi^2$, $\varepsilon_{\text{F}\sigma} = \hbar^2 k_{\text{F}\sigma}^2/2m = \varepsilon_{\text{F}} + \sigma M$, and V_{imp} is the impurity potential specified later (Eq.(12)). The subscript $\sigma = \uparrow, \downarrow$ corresponds, respectively, to $\sigma = +1, -1$ in the formula (and to $\bar{\sigma} = \downarrow, \uparrow$ or $-1, +1$). In the presence of $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}(\mathbf{q}, \omega) e^{i(\mathbf{q} \cdot \mathbf{r} - \omega t)}$, the s -electrons feel a perturbation (note that $H_{\text{el}} + H_{\text{sd}} = \mathcal{H}_0 + \mathcal{H}_1$)

$$\mathcal{H}_1 = -M \sum_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{q}}^{\dagger} \boldsymbol{\sigma} c_{\mathbf{k}} \cdot \mathbf{u}(\mathbf{q}, \omega) e^{-i\omega t}, \quad (7)$$

and acquires a transverse component [22]

$$\langle \hat{\sigma}'_{\perp}(x) \rangle_{\text{n.e.}} = M \int_{-\infty}^t dt' \int d\mathbf{r}' \chi_{\perp}^{\alpha\beta}(x - x') u^{\beta}(x') \quad (8)$$

in their spin polarization, in the first order in \mathbf{u} . Here, $x = (\mathbf{r}, t)$, $x' = (\mathbf{r}', t')$, and $\chi_{\perp}^{\alpha\beta}$ is the transverse spin susceptibility in a uniformly magnetized state. ($\alpha, \beta = x, y$ specify the transverse components and summing over β is implied.) Writing in Fourier components,

$$\langle \hat{\sigma}'_{\perp}(\mathbf{q}, \omega) \rangle_{\text{n.e.}} = M \chi_{\perp}^{\alpha\beta}(\mathbf{q}, \omega + i0) u^{\beta}(\mathbf{q}, \omega), \quad (9)$$

we expand $\chi_{\perp}^{\alpha\beta}(\mathbf{q}, \omega + i0)$ as

$$\begin{aligned} & \chi_{\perp}^{\alpha\beta}(\mathbf{q}, \omega + i0) - \chi_{\perp}^{\alpha\beta}(\mathbf{0}, 0) \\ &= \frac{1}{M^2} [i(\mathbf{b} \cdot \mathbf{q} - b_0 \omega) \delta_{\alpha\beta} + i(\mathbf{a} \cdot \mathbf{q} - a_0 \omega) \varepsilon_{\alpha\beta}] \end{aligned} \quad (10)$$

up to the first order in \mathbf{q} and ω . (Here $\varepsilon_{\alpha\beta}$ is an antisymmetric tensor in 2D, with $\varepsilon_{xy} = 1$.) Below we will see that $M \chi_{\perp}^{\alpha\beta}(\mathbf{0}, 0) = \rho_s \delta_{\alpha\beta}$. Therefore, $\langle \hat{\boldsymbol{\sigma}} \rangle_{\text{n.e.}} = \rho_s \mathbf{n} + \langle \hat{\boldsymbol{\sigma}}_{\perp} \rangle_{\text{n.e.}}$, with $\langle \hat{\boldsymbol{\sigma}}_{\perp} \rangle_{\text{n.e.}}$ given by

$$\frac{1}{M} [b_0 \dot{\mathbf{u}} + (\mathbf{b} \cdot \nabla) \mathbf{u} - a_0 (\hat{z} \times \dot{\mathbf{u}}) - \hat{z} \times (\mathbf{a} \cdot \nabla) \mathbf{u}], \quad (11)$$

which coincides with (5) within the present accuracy (*i.e.*, neglecting $\mathbf{u} \times \dot{\mathbf{u}} = \mathcal{O}(u^2)$), and leads to the spin torque (4). The problem thus reduces to the calculation of the coefficients, a_{μ} and b_{μ} , in Eq.(10) [24].

We consider a 3D electron system with $\varepsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2/2m$, which is affected by the impurity potential represented by

$$V_{\text{imp}} = u \sum_i \delta(\mathbf{r} - \mathbf{R}_i) + u_s \sum_j \mathbf{S}_j \cdot \boldsymbol{\sigma} \delta(\mathbf{r} - \mathbf{R}'_j) \quad (12)$$

in the first-quantization form. The first term describes potential scattering, and the second describes spin scattering by impurity spins, \mathbf{S}_j . We take a quenched average for the impurity spin direction as

$$\overline{S_i^{\alpha} S_j^{\beta}} = \delta_{ij} \delta_{\alpha\beta} \times \begin{cases} \overline{S_z^2} & (\alpha, \beta = x, y) \\ \overline{S_z^2} & (\alpha, \beta = z) \end{cases} \quad (13)$$

as well as for the impurity positions, \mathbf{R}_i and \mathbf{R}'_j , as usual. The electron damping rate is then given by

$$\gamma_{\sigma} = \frac{1}{2\tau_{\sigma}} = \pi n_i u^2 \nu_{\sigma} + \pi n_s u_s^2 (2\overline{S_z^2} \nu_{\bar{\sigma}} + \overline{S_z^2} \nu_{\sigma}) \quad (14)$$

in the first Born approximation, where n_i (n_s) is the concentration of normal (magnetic) impurities, and $\nu_{\sigma} = m k_{\text{F}\sigma}/2\pi^2 \hbar^2$ is the density of states (DOS) at $\varepsilon_{\text{F}\sigma}$. We can include the spin-orbit impurities, of scattering amplitude $i u_{\text{s.o.}} (\mathbf{k} \times \mathbf{k}') \cdot \boldsymbol{\sigma}$, by adding $n_{\text{s.o.}} u_{\text{s.o.}}^2 \overline{(\mathbf{k} \times \mathbf{k}')^2}$ to $n_s u_s^2 \overline{S_z^2}$, in all expressions in the present study.

We assume that $\gamma_{\sigma} \ll \varepsilon_{\text{F}\sigma}$ and $\gamma_{\sigma} \ll M$, and calculate α and β in the lowest non-trivial order in $x_{\sigma} \equiv \gamma_{\sigma}/4\varepsilon_{\text{F}\sigma}$ and γ_{σ}/M , both being collectively denoted as γ . We will see that α and β are quantities of $\mathcal{O}(\gamma)$. For a perturbative estimate in γ , only the first-order correction is sufficient to the σ^{α} - (and σ^{β} -) vertex, since the corresponding ‘diffusion’ ladder has a large mass $\sim M$.

Gilbert damping : We first study the ω -linear terms in the uniform ($\mathbf{q} = \mathbf{0}$) part of the transverse spin susceptibility, $\chi_{\perp}^{\alpha\beta}(\mathbf{q} = \mathbf{0}, \omega + i0)$. Contributions up to $\mathcal{O}(\gamma)$, represented in Fig.1, are given, in Matsubara form, by

$$\chi_{\perp}^{\alpha\beta}(\mathbf{0}, i\omega_{\lambda}) = -T \sum_n \sum_{\sigma} (\delta_{\alpha\beta} + i\sigma \varepsilon_{\alpha\beta}) \varphi_{\sigma}, \quad (15)$$

where

$$\varphi_{\sigma} \equiv \varphi_{\sigma}(i\varepsilon_n; i\omega_{\lambda}) = \bar{\chi}_{\sigma} + \tilde{\Gamma}_0 \bar{\chi}_{\sigma}^2, \quad (16)$$

with $\bar{\chi}_{\sigma} \equiv \bar{\chi}_{\sigma}(i\varepsilon_n; i\omega_{\lambda}) = \sum_{\mathbf{k}} G_{\mathbf{k}\sigma}(i\varepsilon_n + i\omega_{\lambda}) G_{\mathbf{k}\sigma}(i\varepsilon_n)$, $G_{\mathbf{k}\sigma}(z) = (z - \varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{F}\sigma} + i\gamma_{\sigma} \text{sgn}(\text{Im}z))^{-1}$. The vertex correction is associated with a factor $\tilde{\Gamma}_0 = n_i u^2 - n_s u_s^2 \overline{S_z^2}$, where the S_z scattering changes sign whereas S_x and S_y scatterings cancel each other. After analytic continuation ($i\omega_{\lambda} \rightarrow \omega + i0$) and extracting the ω -linear terms, we have

$$b_0 = \frac{M^2 \hbar}{2\pi} \sum_{\sigma} \text{Re}\{\varphi_{\sigma}^{(1)}(0;0) - \varphi_{\sigma}^{(2)}(0;0)\}, \quad (17)$$

at absolute zero, $T = 0$. Here the upper labels, (1) and (2), on φ_{σ} (and $\bar{\chi}_{\sigma}$ below) indicate the analytic continuations, $G(i\varepsilon_n + i\omega_{\lambda})G(i\varepsilon_n) \rightarrow G^R G^R$ and $G^R G^A$, respectively [25].

By an explicit evaluation of the \mathbf{k} -integrals, we have $\bar{\chi}_{\sigma}^{(1)}(0;0) = (i\pi/2M)[\nu_{-} + iy - i(\nu_{-}\gamma_{-}/M)] + \mathcal{O}(\gamma^2)$, $\bar{\chi}_{\sigma}^{(2)}(0;0) = -(i\pi/2M)[\sigma\nu_{+} - iy + i(\nu_{+}\gamma_{+}/M)] + \mathcal{O}(\gamma^2)$, and thus $b_0 = -\hbar(\nu_{+}\gamma_{+} - \nu_{-}\gamma_{-})/4 + (\pi\hbar\tilde{\Gamma}_0/8)(\nu_{+}^2 - \nu_{-}^2)$. Here $\nu_{\pm} = \nu_{\uparrow} \pm \nu_{\downarrow}$, $\gamma_{\pm} = (\gamma_{\uparrow} \pm \gamma_{\downarrow})/2$, and $y = \nu_{\uparrow}x_{\uparrow} - \nu_{\downarrow}x_{\downarrow}$. Using Eq.(14), we finally obtain

$$\alpha = \pi n_s u_s^2 \left[2\overline{S_z^2} \nu_{\uparrow} \nu_{\downarrow} + \overline{S_{\perp}^2} (\nu_{\uparrow}^2 + \nu_{\downarrow}^2) \right] \times \frac{a^3}{S}. \quad (18)$$

As expected, only the spin (and spin-orbit) scattering contributes to α , and the potential scattering ($\sim n_i u^2$) does not thanks to the cancellation between selfenergy and vertex correction [26].

By a similar analysis, we obtain $a_0 = -\hbar\rho_s/2$.

For $i\omega_{\lambda} = 0$, φ_{σ} is independent of σ . Thus, from Eq.(15), $\chi_{\perp}^{\alpha\beta}(\mathbf{0}, 0)$ is proportional to $\delta_{\alpha\beta}$. Explicit calculation shows that $\chi_{\perp}^{\alpha\beta}(\mathbf{0}, 0) = (\rho_s/M) \delta_{\alpha\beta}$ as used above.

β -term : We next examine the \mathbf{q} -linear terms in the presence of current flow under static $\mathbf{n}(\mathbf{r}) = \hat{z} + \mathbf{u}(\mathbf{r})$. We produce a current-carrying state by applying a d.c. electric field \mathbf{E} , and calculate a linear response of σ_{\perp}^{α} to

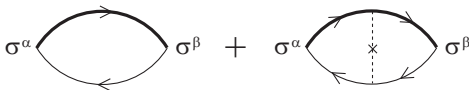


FIG. 1: Transverse spin susceptibility up to $\mathcal{O}(\gamma)$. The dotted line with a cross represents scattering by impurities, either non-magnetic or magnetic (including spin-orbit). The thick (thin) solid line carries Matsubara frequency $i\varepsilon_n + i\omega_{\lambda}$ ($i\varepsilon_n$).

\mathbf{E} . In a similar way to the Kubo formula for electrical conductivity [27], one can derive

$$\langle \hat{\sigma}_{\perp}^{\alpha}(\mathbf{q}) \rangle_{\text{n.e.}} = \chi_i^{\alpha}(\mathbf{q}) E_i, \quad (19)$$

$$\chi_i^{\alpha}(\mathbf{q}) = \lim_{\omega \rightarrow 0} \frac{K_i^{\alpha}(\mathbf{q}, \omega + i0) - K_i^{\alpha}(\mathbf{q}, 0)}{i\omega}, \quad (20)$$

$$K_i^{\alpha}(\mathbf{q}, i\omega_{\lambda}) = \int_0^{\beta} d\tau e^{i\omega_{\lambda}\tau} \langle T_{\tau} \sigma^{\alpha}(\mathbf{q}, \tau) J_i \rangle. \quad (21)$$

Here $\mathbf{J} = -e \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$ is the total current ($\mathbf{v}_{\mathbf{k}} = \hbar \mathbf{k}/m$), and the average $\langle \dots \rangle$ is taken in the thermal equilibrium state determined by $\mathcal{H}_0 + \mathcal{H}_1$ with $\omega = 0$ (in Eq.(7)). To have a non-vanishing contribution, we need to extract u^{β} and q_j both in first order, and put

$$K_i^{\alpha}(\mathbf{q}, i\omega_{\lambda}) = -e M K_{ij}^{\alpha\beta}(i\omega_{\lambda}) q_j u^{\beta}(\mathbf{q}, 0). \quad (22)$$

Diagrammatic expressions for $K_{ij}^{\alpha\beta}$ are given in Fig.2.

The calculation of \mathbf{b} (and \mathbf{a}) as a linear response to \mathbf{E} is similar to the calculation of transport coefficients, but requires a closer analysis to collect all the terms one order higher in γ . The result, up to $\mathcal{O}(\gamma^0)$, is

$$\mathbf{a} = \frac{e\hbar\mathbf{E}}{2m} \sum_{\sigma} \sigma n_{\sigma} \tau_{\sigma} = \frac{\hbar}{2e} \sigma_s \mathbf{E} = \frac{\hbar}{2e} \mathbf{j}_s, \quad (23)$$

$$\mathbf{b} = \frac{e\hbar\mathbf{E}}{2mM} \left[\sum_{\sigma} \sigma n_{\sigma} \tau_{\sigma} \gamma_{\sigma} - \pi \tilde{\Gamma}_0 \sum_{\sigma} \sigma n_{\sigma} \tau_{\sigma} \nu_{\sigma} \right] \quad (24)$$

$$= \frac{\pi n_s u_s^2}{M} \cdot \frac{\hbar}{2e} \left[(\overline{S_{\perp}^2} + \overline{S_z^2}) \nu_{+} \mathbf{j}_s + (\overline{S_{\perp}^2} - \overline{S_z^2}) \nu_{-} \mathbf{j}_c \right], \quad (25)$$

where $\mathbf{j}_s = \sigma_s \mathbf{E} = \mathbf{j}_{\uparrow} - \mathbf{j}_{\downarrow}$ is the spin current, and $\mathbf{j}_c = \sigma_c \mathbf{E} = \mathbf{j}_{\uparrow} + \mathbf{j}_{\downarrow}$ is the charge current, with $\{\sigma_c, \sigma_s\} = (e^2/m)(n_{\uparrow} \tau_{\uparrow} \pm n_{\downarrow} \tau_{\downarrow})$ being charge and spin conductivities. Similarly to the case of α , only the spin (and spin-orbit) scattering contributes to \mathbf{b} [26]. We also see that \mathbf{b} is mainly determined by \mathbf{j}_s , but also depends on \mathbf{j}_c for a general case of $\overline{S_{\perp}^2} \neq \overline{S_z^2}$, and that

$$\beta = \frac{\pi n_s u_s^2}{M} \left[(\overline{S_{\perp}^2} + \overline{S_z^2}) \nu_{+} + \frac{1}{P_j} (\overline{S_{\perp}^2} - \overline{S_z^2}) \nu_{-} \right], \quad (26)$$

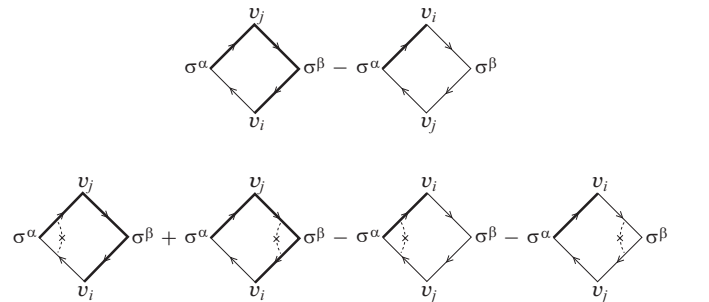


FIG. 2: The E_i -, q_j - and u^{β} -linear coefficient, $K_{ij}^{\alpha\beta}$, of the transverse spin polarization, $\langle \hat{\sigma}_{\perp}^{\alpha}(\mathbf{q}) \rangle_{\text{n.e.}}$, in the presence of current flow (E_i) and magnetization texture ($q_j u^{\beta}$). The velocity vertices with v_i and v_j are associated with E_i and q_j , respectively. Other graphical meanings are the same as Fig.1.

where $P_j = \sigma_s/\sigma_c$ is the polarization of the current.

In terms of longitudinal (τ_L) and transverse (τ_T) spin-relaxation times, obtained as $\tau_L^{-1} = 4\pi n_s u_s^2 S_\perp^2 \nu_+/\hbar$ and $\tau_T^{-1} = 2\pi n_s u_s^2 (\overline{S_\perp^2} + \overline{S_z^2}) \nu_+/\hbar$ [28], we have

$$\alpha = \frac{a^3 \hbar \nu_+}{4S} \left[(1 - P_\nu^2) \frac{1}{\tau_T} + P_\nu^2 \frac{1}{\tau_L} \right], \quad (27)$$

$$\beta = \frac{\hbar}{2M} \left[\left(1 - \frac{P_\nu}{P_j} \right) \frac{1}{\tau_T} + \frac{P_\nu}{P_j} \frac{1}{\tau_L} \right], \quad (28)$$

where $P_\nu = \nu_-/\nu_+$ is the DOS asymmetry. We see that both longitudinal and transverse spin-relaxation processes contribute to α and β in general (*i.e.*, if $P_\nu \neq 0$), in contrast to the demonstration of Ref.[20]. For “isotropic” impurities with $\overline{S_\perp^2} = \overline{S_z^2}$ and thus $\tau_L = \tau_T \equiv \tau_s$, we have

$$\alpha = \frac{a^3 \nu_+}{4S} \cdot \frac{\hbar}{\tau_s}, \quad \beta = \frac{\hbar}{2M \tau_s}. \quad (29)$$

The present results are obtained from those of Zhang and Li [8] if we identify their τ_{sf} and n_0 with our τ_s (or τ_T when $P_\nu = 0$) and $M\nu_+$, respectively.

Finally, we consider an itinerant single-band model such as the Stoner model treated in a mean-field approximation. Suppose we apply a small transverse field, $\mathbf{h} = \mathbf{h}_{\mathbf{q},\omega} e^{i(\mathbf{q}\cdot\mathbf{r} - \omega t)}$, on a uniform ferromagnetic state. The corresponding magnetization dynamics is described by $u^\alpha(\mathbf{q},\omega) = -[\chi_\perp^{\text{RPA}}(\mathbf{q},\omega)]^{\alpha\beta} h_{\mathbf{q},\omega}^\beta$, within linear response. Here $\mathbf{u} \equiv \langle \hat{\boldsymbol{\sigma}}' \rangle_{\text{n.e.}}$ now, and $\chi_\perp^{\text{RPA}} \equiv (\chi_\perp^{-1} - J)^{-1}$ is the RPA (transverse) susceptibility, with χ_\perp given by Eq.(10) and J being the ferromagnetic exchange coupling constant. We write $(\chi_\perp^{\text{RPA}})^{-1} \mathbf{u} = -\mathbf{h}$, and expand as $\chi_\perp^{-1} = \chi_{\perp,0}^{-1} - \chi_{\perp,0}^{-1} \chi_{\perp,1} \chi_{\perp,0}^{-1}$ up to $\mathcal{O}(q,\omega)$, where $\chi_{\perp,n}$ represents terms n -th order in \mathbf{q} and ω . Using the self-consistent equation, $M = J\rho_s$, at equilibrium, we obtain

$$\rho_s^{-2} [b_0 \dot{\mathbf{u}} + (\mathbf{b} \cdot \nabla) \mathbf{u} - \hat{\mathbf{z}} \times (a_0 \dot{\mathbf{u}} + (\mathbf{a} \cdot \nabla) \mathbf{u})] = \mathbf{h}. \quad (30)$$

Within $\mathcal{O}(u)$, this is consistent with the LLG equation

$$\dot{\mathbf{n}} = (\mathbf{a}' \cdot \nabla) \mathbf{n} + \mathbf{n} \times [b'_0 \dot{\mathbf{n}} + (\mathbf{b}' \cdot \nabla) \mathbf{n} - \rho_s^2 \mathbf{h}']. \quad (31)$$

Primed quantities here are those divided by $-a_0 = \hbar\rho_s/2$. This amounts to replacing $2S$ in the localized model by $\rho_s a^3$. For “isotropic” impurities, we have $\alpha = \hbar\nu_+/2\rho_s\tau_s$ and $\beta = \hbar/2M\tau_s$, and thus $\alpha \neq \beta$ even in an itinerant single-band model, in contrast to the result of Ref.[20].

We have presented a model calculation of spin torques, especially the Gilbert damping and the so-called β -term, on the basis of two types of microscopic models for ferromagnetism (localized and itinerant) and a controlled approximation. Magnetic (and spin-orbit) impurities have been used as a model for spin relaxation in conducting electron systems. In actual systems, however, the origin of spin relaxation will be various depending on specific systems. We hope the present formalism will be useful for subsequent studies treating such more realistic models.

We would like to thank Y. Nakatani, H. Ohno, T. Ono and Y. Suzuki for valuable discussions. This work is financially supported by Monka-sho, Japan.

* Electronic address: kohno@mp.es.osaka-u.ac.jp

- [1] J.C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
- [2] L. Berger, Phys. Rev. **B 54**, 9353 (1996).
- [3] Ya. B. Bazaliy, B. A. Jones, and S.-C. Zhang, Phys. Rev. **B 57**, R3213 (1998).
- [4] J.-Ph. Ansermet, IEEE Trans. Magn. **40**, 358 (2004).
- [5] J. Fernández-Rossier, M. Braun, A.S. Núñez and A. H. MacDonald, Phys. Rev. **B 69**, 174412 (2004).
- [6] Z. Li and S. Zhang, Phys. Rev. Lett. **92**, 207203 (2004).
- [7] J. Shibata, G. Tatara and H. Kohno, Phys. Rev. Lett. **94**, 076601 (2005), and unpublished note.
- [8] S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004).
- [9] A. Thiaville, Y. Nakatani, J. Miltat and Y. Suzuki, Europhys. Lett. **69**, 990 (2005).
- [10] S.E. Barnes and S. Maekawa, Phys. Rev. Lett. **95**, 107204 (2005).
- [11] G. Tatara, T. Takayama, H. Kohno, J. Shibata, Y. Nakatani and H. Fukuyama, to appear in, J. Phys. Soc. Jpn. (2006).
- [12] C. Heide, P.E. Zilberman and R.J. Elliott, Phys. Rev. **B 63**, 064424 (2001).
- [13] Y. Tserkovnyak, A. Brataas and G.E.W. Bauer and B.I. Halperin, Rev. Mod. Phys. **77**, 1375 (2005).
- [14] L. Berger, J. Appl. Phys. **55**, 1954 (1984).
- [15] X. Waintal and M. Viret, Europhys. Lett. **65**, 427 (2004).
- [16] G. Tatara and H. Kohno, Phys. Rev. Lett. **92**, 086601 (2004).
- [17] D.M. Edwards, F. Federici, J. Mathon and A. Umerski, Phys. Rev. **B 71**, 054407 (2005).
- [18] M. Onoda and N. Nagaosa, Phys. Rev. Lett. **96**, 066603 (2006).
- [19] H. Kohno, G. Tatara and J. Shibata, unpublished note.
- [20] Y. Tserkovnyak, A. Brataas and G.E.W. Bauer, cond-mat/0512715.
- [21] Please note that the direction of \mathbf{n} is opposite to the magnetization, and the direction of electron flow is opposite to that of spin and charge currents, \mathbf{j}_s and \mathbf{j}_c , which are defined just after Eq.(25).
- [22] We define $\hat{\boldsymbol{\sigma}}_\perp = \hat{\boldsymbol{\sigma}} - \mathbf{n}(\mathbf{n} \cdot \hat{\boldsymbol{\sigma}})$ and $\hat{\boldsymbol{\sigma}}'_\perp = \hat{\boldsymbol{\sigma}} - \hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot \hat{\boldsymbol{\sigma}})$. Note that $\langle \hat{\boldsymbol{\sigma}} \rangle_{\text{n.e.}}$ in Eq.(3) can be replaced by $\langle \hat{\boldsymbol{\sigma}}_\perp \rangle_{\text{n.e.}}$.
- [23] Y. Tserkovnyak, G.A. Fiete and B.I. Halperin, Appl. Phys. Lett. **84**, 5234 (2004).
- [24] Similar analysis on the Gilbert damping has been done in, A. Sakuma, cond-mat/0602075.
- [25] In deriving Eq.(17), we have noted that, under $\sigma \rightarrow -\sigma$, $\varphi_\sigma^{(1)}(0;0)$ and $\text{Re} \varphi_\sigma^{(2)}(0;0)$ are even, whereas $\text{Im} \varphi_\sigma^{(2)}(0;0)$ and $\partial \varphi_\sigma^{(1)}(\varepsilon;\omega)/\partial \omega|_{\omega=0}$ are odd.
- [26] Note that spin-orbit coupling in the bulk band structure is not taken into account in the present study.
- [27] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
- [28] These expressions, obtained from ‘diffusion’ ladder diagrams, also agree with our intuition; for example, transverse relaxation, namely, spin flip of, *e.g.*, σ_x -component, is induced by S_y - and S_z -scatterings, hence is associated with a factor, $|S_y + iS_z|^2 = \overline{S_\perp^2} + \overline{S_z^2}$.